# **SECTION-A**

# **UNIT I: MICROWAVE TUBES**

# Lecture 07

# **Limitations of Conventional Vacuum Tubes**

Conventional vacuum triodes, tetrodes and pentodes are less useful signal sources at frequencies above 1GHz because of lead inductance and inter-electrode-capacitance effects, transit angle effects and gain-bandwidth product limitations. These three effects are analysed in the following section:

- Lead inductance and inter-electrode capacitance effects
- Transit angle effects
- Gain bandwidth limitation

#### I. Lead Inductance and Inter-electrode capacitance effects.



Fig.(i): Triode CircuitFig.(ii): Equivalent of Triode circuit

We assume that following discussion will be carried out on two assumptions

i.  $C_{gp} \ll C_{gk}$ ii.  $\omega L_k \ll \frac{1}{\omega C_{gk}}$ 

For V<sub>in</sub>

$$\therefore V_{in} = V_g + V_k \therefore V_k = IR = g_m V_g X_L \therefore V_{in} = V_g + j\omega L_k g_m V_g \qquad (1)$$

For *I*<sub>in</sub>

$$\therefore I_{in} = \frac{V}{R} = \frac{V_g}{X_c} = \frac{V_g}{\frac{1}{j\omega C_{gk}}}$$

$$\therefore I_{in} = j\omega V_g C_{gk} \dots (2)$$

From equation (1)

$$\therefore V_{in} = (1 + j\omega L_k g_m) V_g \dots (3)$$

From equation (3)

$$\therefore V_g = \frac{I_{in}}{j\omega C_{gk}}$$

Equation (3) becomes

$$\therefore V_{in} = \frac{I_{in}}{j\omega C_{gk}} (1 + j\omega L_k g_m)$$

For  $Y_{in}$ 

$$\therefore Y_{in} = \frac{I_{in}(j\omega C_{gk})}{I_{in}(1+j\omega L_k g_m)}$$
$$\therefore Y_{in} = (j\omega C_{gk}) \frac{1}{(1+j\omega L_k g_m)}$$

Using for  $x \ll 1$ 

For  $Z_{in}$ 

$$\therefore Z_{in} = \frac{1}{\omega^2 L_k C_{gk} g_m + j \omega C_{gk}}$$
$$\therefore Z_{in} = \frac{(\omega^2 L_k C_{gk} g_m - j \omega C_{gk})}{(\omega^2 L_k C_{gk} g_m + j \omega C_{gk})(\omega^2 L_k C_{gk} g_m - j \omega C_{gk})}$$

$$\therefore Z_{in} = \frac{\omega^2 L_k C_{gk} g_m}{\omega^2 L_k C_{gk} g_m + j \omega C_{gk}} - \frac{j \omega C_{gk}}{\omega^2 L_k C_{gk} g_m - j \omega C_{gk}}$$

$$\therefore Z_{in} = \frac{1}{\omega^2 L_k C_{gk} g_m} - j \frac{1}{\omega^3 L_k C_{gk} g_m^2}$$

This is output impedance of triode circuit considering lead inductance and inter-electrode capacitance effect.

The real part of impedance is inversely proportional to the square of frequency and imaginary part is inversely proportional to third order of frequency.

When frequencies are above 1GHz, the real part of the impedance becomes small enough to nearly short the signal source. Consequently, the output power is decreased rapidly.

There are several ways to minimize the inductance and capacitance effects, such as a reduction in lead length and electrode area ( $L_k$  and  $C_{gk}$ ). This minimization, however also limits the power handling capacity.

### II. <u>Transit Angle Effects</u>

$$\theta_g = \omega \tau_g$$
$$\therefore \ \theta_g = \frac{\omega d}{v_0}$$

Where,

 $\omega$  = Angular frequency d = Separation between cathode and grid  $v_0$  = Velocity of electron  $\theta_g$  = Transit angle  $\tau_g$  = Transit time

Due to this effect efficiency of conventional vacuum tube reduces at high frequency.

When frequency is below microwave range the transit, angle is negligible. At microwave the transit is large compared to the period of microwave signal, the potential between the cathode and grid may alternate from 10 to 100 times during negative half cycle thus removes energy that was given to electron during the positive half cycle. The overall result of transit angle effects is to reduce the operating efficiency of vacuum tubes. The degenerate effect becomes more serious when frequency is well about 1GHZ. Once electrons pass the grid, they are quickly accelerated to the anode by the high plate voltage.

From preceding analysis, it is clear that transit angle effect can be minimized by first accelerating the electron beam with a very high dc voltage and then velocity modulating it. And this is indeed the principle of operation of such microwave tubes such as klystron and magnetrons.

### I. Gain Bandwidth Product Limitation

(Due to this effect higher gain is achieved only at the expense of narrower bandwidth.)



Fig. (3): Output tuned circuit of pentode

The load voltage is given by

Maximum gain is possible at resonant frequency

$$\therefore f_r = \frac{1}{2\pi\sqrt{Lc}}$$
$$\therefore A_{max} = \frac{\frac{V_l}{V_g}}{\frac{g_m}{G}}$$

Since the bandwidth is measured at half power point

$$\therefore A_{max} = \frac{g_m}{2G}$$

The denominator of equation (1) must be related by

$$\therefore G = \omega c - \frac{1}{\omega L}$$
$$\therefore \omega GL = \omega^2 cL - 1$$

$$\therefore \ \omega^2 cL \ - \ \omega GL \ -1 = 0$$

To find roots of above quadratic equation consider

$$a = cL, b = -G, c = -1$$

$$\therefore \omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\omega = \frac{GL \pm \sqrt{G^2 L^2 + 4cL}}{2cL}$$

$$\therefore \omega = \frac{G \pm \sqrt{G^2 + \frac{4c}{L}}}{2c}$$
$$\therefore \omega = \frac{G}{2c} \pm \sqrt{\left(\frac{G}{2c}\right)^2 + \frac{1}{Lc}}$$

$$\omega_1 = \frac{G}{2c} - \sqrt{\left(\frac{G}{2c}\right)^2 + \frac{1}{Lc}}$$

$$\omega_2 = \frac{G}{2c} + \sqrt{\left(\frac{G}{2c}\right)^2 + \frac{1}{Lc}}$$

Now Bandwidth can be expressed by

$$BW = \omega_2 - \omega_1$$
$$BW = \frac{G}{2c} + \frac{G}{2c} - \frac{G}{2c} + \frac{G}{2c} = \frac{G}{c} \qquad \dots \qquad \left(\frac{G}{2c}\right)^2 \gg \frac{1}{L}$$

Now the gain-bandwidth product is

$$A_{max}BW = \frac{g_m}{G} * \frac{G}{c} = \frac{g_m}{c}$$
$$A_{max} = \frac{g_m}{c * BW}$$

$$\therefore A_{max} \propto \frac{1}{BW}$$

This shows that  $A_{max}$  is inversely proportional to bandwidth.

It is important to note that the gain bandwidth product is independent of frequency. For a given tube a higher gain can be achieved only at the expense of the narrower bandwidth. This restriction is applicable to a resonant circuit only. In microwave devices either re-entrant cavities or slow wave structures are used to obtain a possible overall high gain over broad bandwidth.

#### **Some frequently asked university exam questions:**

1. Explain the limitations of conventional devices at microwave frequency?

(W-16, S-15, S-14)

## Lecture 09

# **Diagram of Two cavity klystron Amplifier:**



Fig. (4.1): Two cavity klystron Amplifier:

### **Constructional Details:**

A klystron amplifier uses different cavities which control the electric field around the axis of the tube. A grid is placed in the middle of these cavities to allow the electrons to flow. The first cavity together with the coupling device is termed as a 'Buncher'. In the same way, the second cavity with coupling device is called as a 'Catcher'. The field direction changes with the frequency of the Buncher cavity. In above diagram coaxial cavity is used. Several types of reentrant cavities are shown in fig1.b





## **Operation:**

- It is widely used microwave amplifier operated by principles of velocity and current modulation.
- Assume that input signal is smaller than DC accelerating voltage.
- All electrons injected from the cathode arrive at the first cavity with uniform velocity.
- Those electrons passing the first cavity gap at zero gap voltage passes through without change in velocity.
- Those passing through a positive half cycle of gap voltage passes with increase invelocity.
- And those passing through the negative half cycle of gap voltage passes with deceasing invelocity.
- Above action results the variations in electron velocity gradually bunch together when those travel down the drift space.
- And the variations in electron velocity in the drift space are known as velocity modulation.
- The density of electrons in the second cavity gap varies cyclically with time.
- The electron beam contains an AC component and is said to be current modulated. The drift space converts the velocity modulation into current modulation.
- With proper design (optimum gap widths, anode to cathode voltage, drift space length etc.), a little RF power applied to the buncher cavity results in large beam currents at the cathcher cavity with a considerable power gain. And hence when V2 is exceed than V1, then amplification takes place.

## **Performance Characteristics:**

- 1. Frequency: 250 MHz to 100 GHz.
- 2. Power: 10kw-500kw (CW) 30Mw (Pulsed)
- 3. Power gain: 15dB to 70dB (60 dB nominal)

- 4. Bandwidth: Limited (Because cavity resonator is used) 10-60MHz- generally used in fixed frequency applications.
- 5. Noise Figure: 15-20 dB (Sometimes greater than 25 dB)
- 6. Theoretical efficiency: 58% (30-40% nominal)

## **Applications:**

- 1. As a power output tube
- > In UHF TV transmitters
- ➢ In troposphere scatter transmitters
- Satellite communication and ground stations
- Radar transmitters
- 2. As a power oscillator.

# Lecture 10 BUNCHING PROCESS:

Once the electrons leave the buncher cavity, they drift with a velocity given by Eq.(1) or (2) along in the field-free space between the two cavities. The effect of velocity modulation produces bunching of the electron beam – or current modulation. The electrons that pass the buncher  $asV_s = 0$  travel through with unchanged velocity  $v_o$  and become the bunchingcenter. Those electrons that pass the buncher cavity during the positive half cycles of the microwave input voltage  $V_s$  travel faster than the electrons that passed the gap when  $V_s = 0$ . Those electrons that pass the buncher cavity during the negative half cycles of the microwave input voltage  $V_s$  travel slower than the electrons that passed the gap when  $V_s = 0$ . At a distance of  $\Delta L$  along the beam from the bunchercavity, the beam electrons have drifted into dense clusters. Fig(1) shows the trajectories of minimum, zero, and maximum electron acceleration.

Eq. (1) is the equation of velocity modulation which is given as

 $v(t_1) = v_0 \left[ 1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \right]$  .....(2)

Alternatively, the equation of velocity modulation can be given by





The distance from the buncher grid to the location of dense electron bunching for the electron at  $t_b$  is

$$\Delta L = v_o(t_d - t_b) \qquad \dots \dots (3)$$

Similarly, the distance for the electrons at  $t_d$  and  $t_c$  are

$$\Delta L = v_{min}(t_d - t_a) = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \qquad \dots \dots (4)$$
$$\Delta L = v_{max}(t_d - t_c) = v_{max}\left(t_d - b - \frac{\pi}{2\omega}\right) \qquad \dots \dots (5)$$

From Eq. (1) or (2) the minimum and maximum velocities are

Substitution of eq. (6) and (7) in eq. (4) and (5), respectively, yields the distance

$$\Delta L = v_o(t_d - t_b) + \left[ v_o \frac{\pi}{2\omega} - v_o \frac{\beta_i V_1}{2V_0} (t_d - t_b) - t_o \frac{\beta_i V_1}{2V_0} \frac{\pi}{2\omega} \right] \dots \dots (8)$$

And

$$\Delta L = v_o(t_d - t_b) + \left[ -v_o \frac{\pi}{2\omega} + v_o \frac{\beta_i V_1}{2V_0} (t_d - t_b) + t_o \frac{\beta_i V_1}{2V_0} \frac{\pi}{2\omega} \right] \quad \dots \dots \dots (9)$$

The necessary condition for those electrons at  $t_a$ ,  $t_b$  and  $t_c$  to meet at the same distance  $\Delta L$  is

And

Consequently,

And From eq.(10)

It should be noted that the mutual repulsion of the space charge is neglected, but the qualitative results are similar to the preceding representation when the effects of repulsion are included. Furthermore, the distance given by eq.(13) is not the one for a maximum degree of bunching. What should the spacing between the buncher and catcher cavities in order to achieve a maximum degree of bunching? Since the drift region is field free, the transit time for an electron to travel a distance of L is given by

Where the binomial expansion of  $(1 + x)^{-1}$  for |x| << 1 has been replaced  $T_0 = \frac{L}{v_0}$  is the dc transit time. In terms of radians the preceding expression can be written

$$T = \omega T_0 - \omega T_0 \frac{\beta_i V_1}{2V_o} \theta_o \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \qquad \dots \dots \dots (15)$$
$$T = \theta_o - \frac{\beta_i V_1}{2V_o} \theta_o \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \dots \dots \dots \dots (16)$$
$$\pi N \qquad \dots (17)$$

Where,  $\theta_0 = \frac{\omega L}{v_o} = 2\pi N$  ...

Is the transit angle between cavities, N is the number of electron transit cycles in the drift space, and?

It is defined as the bunching parameter of a klystron.

$$\omega T = \theta_o - \operatorname{Xsin}\left(\omega t_1 - \frac{\theta_g}{2}\right)$$
$$\theta_o = \omega T_0 = \frac{\omega L}{v_o}$$
$$L = \frac{\theta_o v_o}{\omega}$$

From eq.(18)

$$\theta_o = \frac{2V_o X}{\beta_i V_1}$$
$$X=1.841$$

The optimum distance L at which the maximum fundamental component of current occurs is computed as

$$L_{optimum} = \frac{3.682\nu_0 V_0}{\omega\beta_i V_1} \qquad \dots \dots \dots (19)$$



It is interesting to note that the distance given by eq.(13) is approx. 15% less than the result of eq.(14). The discrepancy is due in part to the approximation made in deriving eq.(13) and to the fact that the maximum fundamental component of current will not coincide with the maximum electron density along the beam because the harmonic components exist in the beam.

## **Output Power and Efficiency of TCKA:**

#### **Output Power:**

The maximum bunching should occurapproximately midway between the catcher grids. The phase of the catcher gap voltage must be maintained in such a way that the bunched electrons, as they pass through the grids, encounter a retarding phase. When the bunched electron beam passes through the retarding phase, its kinetic energy is transferred to the field of the catcher cavity. When the electrons emerge from the catcher grids, they have reduced velocity and finally collected by the collector.

Total current induced in the catcher cavity is,

$$i_2 = I_0 + \sum_{0}^{\infty} 2I_0 J_n(nx) \cos[n\omega(t_2 - \tau - t_0)]$$

Since the current induced by the electron beam in the walls of the catcher cavity is directly proportional to the amplitude of the microwave input voltage V1, the fundamental component of current induced microwave current in the catcher cavity is given by,here n=1 fundamental frequency

$$i_{2ind} = \beta_0 2I_0 J 1(X) \cos[\omega(T2 - \tau - T0)].....(1)$$

where  $\beta 0$  is the beam coupling coefficient of the catcher gap. If the buncher and catcher cavities are identical, then  $\beta i=\beta 0$ . The fundamental component of current induced in the catcher cavity then has a magnitude

$$i_{2ind} = \beta_0 I_2 = \beta_0 2 I_2 J1(X)....(2)$$

Fig. 6 shows an output equivalent circuit in which  $R_{sho}$  represents the wall resistance of catcher cavity,  $R_B$  the beam loading resistance,  $R_L$  the external load resistance, and Rsh the effective shunt resistance.



Fig.6 Output equivalent circuit

The output Power delivered to the catcher cavity and load is given as,

Pac =  $I^2 \mathbf{R} = (\frac{\beta_0 I_2}{\sqrt{2}})^2 R_{sh}$  .....(3)

Also,  $P_{ac=I_2V_2}P_{ac} = \frac{\beta_0 I_2 V_2}{\sqrt{2}\sqrt{2}}$ 

Where Rsh is the total equivalent shunt resistance of the catcher circuit, including the load, and V2 is the fundamental component of the catcher gap voltage.

#### **Efficiency of Klystron:**

The electronic efficiency of the klystron amplifier is defined as the ratio of the output power to the input power:

.....(5)

Put  $I_2 = 2I_0 J1(X)$ 

Efficiency= $\frac{\beta_0 J_1(X)V_2}{V_0} \quad \dots \quad (6)$ 

In which the power losses to the beam loading and cavity walls are included.

If the coupling is perfect,  $\beta_0=1$ , the maximum beam current approaches I2max =  $2I_0(0.582)$ , and voltage V2 is equal to V0. Then the maximum electronic efficiency is about 58%. In practice, the electronic efficiency of a klystron amplifier is in range of 15 to 30%. Since the efficiency is a function of the catcher gap transit angle  $\theta_g$ . fig.6 shows the maximum efficiency of a klystron as a function of catcher transit angle.

Put above value in equation (4) and equation become,

Efficiency=
$$\frac{1*0.582*V_0}{V_0}$$

$$\eta = 58.2\%$$



Fig 7. Maximum efficiency of klystron verses transit angle.

## **Mutual Conductance and Voltage Gain of Two Cavity Klystron Amplifier:**

**Mutual Conductance of Two Cavity klystron amplifiers:**The equivalent mutual conductance of the klystron amplifier can be defined as the ratio of the induced output current to input voltage. That is,

$$|G_m| = \frac{i_{2ind}}{v_1} \tag{1}$$

Put  $i_{2ind} = \beta_0 I_2$ 

The input voltage V1 can be expressed in terms of bunching parameter X as

$$V_{1=\frac{2XV_0}{\theta_{0\beta_i}}}$$
 And  $X=\frac{\beta_i V_1 \theta_0}{2V_0}$ 

Then equation (2) becomes,

$$|G_m| = \frac{2J_1(X)\beta_0 I_0 \theta_0 \beta_i}{2V_0 X} \dots \dots \dots (3)$$
  
$$G_{0=\frac{I_0}{V_0}} \qquad \text{It is assumed that } \beta 0 = \beta i.$$

Then normalized mutual conductance as,

$$|G_m| = \beta_0^2 \theta_0 G_0 \frac{J_1(X)}{X}$$

$$\frac{|G_m|}{G_0} = \beta_0^2 \theta_0 \frac{J_1(X)}{X}$$

.....(4)

Where  $G_{0=\frac{I_0}{V_0}}$  is the dc beam conductance? The mutual conductance is not a constant but decreases as the bunching parameter X increases. Fig.7 shows the curves of normalized Transconductance as a function of X.



Fig.1 Normalized transconductance versus bunching parameter.

It can be seen from the curves that, for a small signal, the normalized transconductance is maximum. That is,

$$\frac{|G_m|}{G_0} = \frac{\beta_0^2 U_0}{2}$$

For the maximum output at X = 1.841, the normalized mutual conductance is

## **Voltage Gain:**

The voltage gain of a klystron amplifier is defined as

$$A_{v=\left|\frac{V_2}{V_1}\right|=\frac{\beta_0 I_2 R_{sh}}{V_1}}\dots\dots\dots(6)$$

Put  $V_{1=\frac{2XV_0}{\theta_0\beta_i}}$  and  $\beta_0 I_{2=2\beta_0 I_0 J_1(X)}$ 

Where,

$$G_{0=\frac{I_0}{V_0}}$$
 Is the beam conductance

Now equation (6) becomes

$$A_{\nu=\beta_0^2 G_0 R_{sh} \frac{J_1(X)}{X}}....(7)$$
$$A_{\nu} = G_m R_{sh}$$

.....(8)

# **Numerical-TCKA:**

- <u>**Q1**</u>. Two cavity Klystron amplifier has a following parameters  $V_0 = 1000V$ ,  $R_0 = 40k\Omega$ ,  $I_0 = 25mA$ , f = 3GHz, gap spacing in either cavity d = 1mm, spacing between 2 cavities L = 4cm, effective shunt impedance excluding beam loading  $R_{sh} = 30K\Omega$ .
  - a) Find the input gap voltage to give maximum voltage  $V_2$ .
  - b) Find the voltage gain neglecting the beam loading in the output cavity.
  - c) Find the efficiency of amplifier neglecting the beam loading.
  - d) Calculate the beam loading conductance and show that neglecting it was justifying in the preceding calculations.

## **Solution:**

Given: V<sub>0</sub>= 1000V,

- $R_0 = 40K\Omega$ ,  $I_0 = 25mA$ , f = 3GHz, d = 1mm, L = 4cm,
- R<sub>sh</sub>= 30KΩ.
  a) For maximum voltage V<sub>2</sub>,J<sub>1</sub>(X) must be maximum. This means J<sub>1</sub>(X) = 0.582at X = 1.841. Electron velocity just leaving thecathode is,

$$v_0 = 0.593 \times 10^6 \times \sqrt{V_0}$$
  
 $v_0 = 0.593 \times 10^6 \times \sqrt{1000}$ 

$$v_0 = 18.75 \times 10^6 \text{m/s}$$

The gap transit angle is,

$$\theta_g = \omega \frac{d}{v_0} = 2\pi (3 \times 10^9) \frac{10^{-3}}{1.88 \times 10^7} = 1 \text{ rad}$$

The beam-coupling coefficient is,

$$\beta_i = \beta_0 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \frac{\sin(1/2)}{1/2} = 0.952$$

The dc transit angle between the cavities is,  

$$\theta_0 = \omega T_0 = \omega \frac{L}{v_0} = 2\pi (3 \times 10^9) \frac{4 \times 10^{-2}}{1.88 \times 10^7} = 40 \text{ rad}$$

The maximum input voltage  $V_1$  is then given by,

$$V_{1 max} = \frac{2V_0 X}{\beta_i \theta_0} = \frac{2(10^3)(1.841)}{(0.952)(40)} = 96.5 \text{ V}$$

**b**) The voltage gain is found as,

$$A_{\rm v} = \frac{\beta_0^2 \theta_0 J_1(X)}{R_0 X} R_{sh} = \frac{(0.952)^2 (40)(0.582)(30 \times 10^3)}{4 \times 10^4 \times 1.841}$$
  
= 8.959

c) The efficiency can be found as follows,  

$$I_2 = 2I_0 J_1(X) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1 \times 10^{-3} \text{ A}$$
  
 $V_2 = \beta_0 I_2 R_{sh} = (0.952)(29.1 \times 10^{-3})(30 \times 10^3) = 831 \text{ V}$ 

Efficiency = 
$$\frac{\beta_0 I_2 V_2}{2I_0 V_0} = \frac{(0.952)(29.1 \times 10^{-3})(831)}{2(25 \times 10^{-3})(10^3)} = 46.2\%$$
  
**d**) The beam loading conductance  $G_B$  is,  
 $G_B = \frac{G_0}{2} \left( \beta_0^2 - \beta_0 \cos \frac{\theta_g}{2} \right)$   
 $G_B = \frac{25 \times 10^6}{2} [(0.952)^2 - (0.952) \cos(28.6^0)]$   
 $G_B = 8.8 \times 10^{-7}$  mho

Then the beam loading resistance  $R_B$  is,

$$R_B = \frac{1}{G_B} = 1.14 \times 10^6 \,\Omega$$

In comparison with  $R_L$  and  $R_{sho}$  or the effective shunt resistance  $R_{sh}$ , the beam loading resistance is like an open circuit and thus can be neglected in the preceding calculations.

**Q2**. A two cavity Klystron amplifier operates at 5 GHz with a DC beam voltage of 10 KV and a 2 mm cavity gap. For a given input RF voltage, the magnitude of the gap voltage is 100 V. Calculate the transient time at the cavity gap, the transient angle and the velocity of the electrons leaving the gap.

## Solution:

Given:  $V_0 = 10KV$ ,  $V_1 = 100V$ , f = 5GHz, d = 2mm.

$$v_0 = 0.593 \times 10^6 \times \sqrt{V_0}$$
  
 $v_0 = 0.593 \times 10^6 \times \sqrt{10 \times 10^3}$   
 $v_0 = 0.593 \times 10^8 \text{m/s}$ 

a) The transient time at the cavity gap is,

 $T_g = t_1 - t_0 = \frac{d}{v_0} = \frac{2 \times 10^{-3}}{0.593 \times 10^8} = 33.73 \times 10^{-12} \text{ sec} = 33.73 \text{ psec}$ 

**b**) The transit angle is,

$$\theta_g = \omega T_g = \frac{\omega d}{v_0} = 2\pi (5 \times 10^9) \times 33.73 \times 10^{-12}$$
  
 $\theta_g = 1.059 \text{ rad} = 60.7 \text{ deg}$ 

c) For velocity of electrons leaving the gap,

$$v(t_1) = v_0 \left[ 1 + \frac{\beta_1 v_1}{2v_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \right]$$
  

$$v_{\min} = v_0 \left[ 1 - \frac{\beta_1 v_1}{2v_0} \right]$$
  

$$= 0.593 \times 10^8 \left[ 1 - \frac{0.95 \times 100}{2 \times 10^4} \right]$$
  

$$v_{\min} = 59.02 \times 10^6 \text{ m/s} = 0.5902 \times 10^8 \text{ m/s}$$
  

$$v_{\max} = 59.58 \times 10^6 \text{ m/s} = 0.5958 \times 10^8 \text{ m/s}$$

Numerical, Multi-cavity Klystron Amplifier

# Lecture 14 <u>REFLEX KLYSTRONS:</u>



Figure 8.1 Schematic diagram of a reflex klystron

- If a fraction of the output power is fed back to the input cavity and if the loop gain has a magnitude of unity with a phase shift of multiple 2π, the klystron will oscillate.
- We have studied a two-cavity klystron oscillator is usually not constructed because, when the oscillation frequency is varied, the resonant frequency of each cavity and the feedback path phase shift must be readjusted for a positive feedback.

• The reflex klystron is a single-cavity klystron that overcomes the disadvantages of the two-cavity klystron oscillator.

Where,

 $t_0$  = time for electron entering cavity gap at z = 0

 $t_1$  = time for same electron leaving cavity gap at z = d

 $t_2$  = time for same electron returned by retarding field z = d and collected on wall of cavity



Fig 8.2: Applegate diagram with gap voltage for a reflex klystron

### Working of reflex klystron:

- The electron beam injected from the cathode is first velocity-modulated by the cavity-gap voltage.
- Some electrons accelerated by the accelerating field enter the repeller space with greater velocity than those with unchanged velocity. Some electrons decelerated by the retarding field enter the repeller region with less velocity.

- All electrons turned around by the repelled voltage then pass through the cavity gap in bunches that occur once per cycle.
- On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field and give up their kinetic energy to the electromagnetic energy of the field in the cavity.
- Oscillator output energy is then taken from the cavity. The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.

### **Characteristics:**

- Frequency range 2-200 GHz
- Bandwidth 30MHz
- Power Output 10mW-2.5W
- Efficiency 20% to 30%.

### **Applications:**

- Signal source in microwave generator.
- Local oscillators in receivers.
- Pump oscillators in parametric amplifier.
- FM oscillator in low power microwave links.

## **REFLEX KLYSRON VELOCITY MODULATION:**

1. Velocity of electrons before entering cavity

$$\vartheta_0 = 0.593 \times 10^6 \sqrt{\vartheta_0}$$

Velocity of electrons when they are exited from cavity at  $t = t_1$ 

V (t<sub>1</sub>) = 
$$\vartheta_0 [1 + \frac{\beta_i}{2\vartheta_0} \sin(\omega t_1 - \frac{\theta_g}{2})]$$

Analysis of reflex klystron (RK) is similar to two cavity klystron (TCK)

Electric field present in the require space.

$$\mathbf{E} = \frac{V_r + V_0 + V_1 \, \sin(\omega t)}{L}$$

Force on electron in an electric field.

F = -e. E

We know that, F = m.a

$$\mathbf{a} = \frac{\partial^2 z}{\partial t^2} = \frac{-e}{m} \mathbf{E}_z$$

$$\mathbf{M}.\frac{\partial^2 z}{\partial t^2} = -\mathbf{e}. \mathbf{E}$$

$$\mathbf{m} \cdot \frac{\partial^2 z}{\partial t^2} = \frac{-e \left(V_r + V_0\right)}{mL} \qquad \dots V_1 << V_0$$

Integrating,

$$\frac{\partial z}{\partial t} = \frac{-e \left( V_r + V_0 \right)}{mL} \int_{t1}^t dt$$

$$\frac{\partial z}{\partial t} = \frac{-e \left(V_r + V_0\right)}{mL} \left(t - t_1\right) + k_1$$

At 
$$t = t_1, \frac{\partial z}{\partial t} = \vartheta(t_1) = k_1$$

$$\frac{\partial z}{\partial t} = \frac{-e \left(V_r + V_0\right)}{mL} \left(t - t1\right) + \vartheta(t_1)$$

Integrating,

$$\frac{\partial z}{\partial t} = \frac{-e \left(V_r + V_0\right)}{mL} \left[ \int_{t_1}^t (t - t_1) dt \right] + \vartheta(t_1) \int_{t_1}^t dt$$
$$= \frac{-e \left(V_r + V_0\right)}{mL} \left[ \frac{\left((t - t_1)^2\right)}{2} \right] + \vartheta(t_1) \left(t - t_1\right) + k_2$$

At  $t=t_1$ , z=d=K2

$$Z = \frac{-e (V_r + V_0)}{2mL} (t - t1)^2 + \vartheta(t_1) (t - t_1) + d$$

$$d = \frac{-e (V_r + V_0)}{2mL} (t - t1)^2 + \vartheta(t_1) (t - t_1)$$
$$\frac{-e (V_r + V_0)}{2mL} (t - t1)^2 + \vartheta(t_1) (t - t_1) = 0$$

 $\therefore$ At t=t2; z=d,

$$\frac{-e(V_r + V_0)}{2mL}(t_2 - t_1)^2 + \vartheta(t_1)(t_2 - t_1) = 0$$
$$\frac{e(V_r + V_0)}{2mL}(t_2 - t_1) = \vartheta(t_1)$$

Now roundtrip time will be

$$\therefore \mathbf{T}' = (\mathbf{t}_2 - \mathbf{t}_1) = \frac{\vartheta(\mathbf{t}_1) 2mL}{e(V_r + V_0)}$$
$$\mathbf{V}(\mathbf{t}_1) = \vartheta_0 \left[1 + \frac{\beta_i V_1}{2\vartheta_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$
$$\mathbf{T}' = (\mathbf{t}_2 - \mathbf{t}_1) = \frac{2mL}{e(V_r + V_0)} \vartheta_0 \left[1 + \frac{\beta_i V_1}{2\vartheta_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

Multiply above equation by the angular frequency  $(\omega)$  to get around trip transient angle.

$$T' = \omega(t_2 - t_1) = \omega T'_0 [1 + \frac{\beta_i V_1}{2\theta_0} \sin(\omega t_1 - \frac{\theta_g}{2})]$$

Where

$$\mathbf{T}'_{0} = \frac{2mL}{e(V_{r}+V_{0})} \,\vartheta_{0}$$

$$\omega \mathbf{T} = \theta'_0 + X' \sin(\omega t_1 - \frac{\theta_g}{2})$$

$$\theta'_0 = \omega T'_0$$
 and  $X' = \frac{\beta_i V_1 \theta'_0}{2 \vartheta_0}$ 

Where

- *X'* Bunching parameter of reflex klystron.
- $\theta'_0$  round trip D.C. transient angle of the center of the bunch parameter.

**Output power and efficiency of Reflex Klystron Oscillator:** 

### Numerical on reflex klystron-

**Que.** 1) Reflex klystron operated at the peak n=2 with a beam voltage 300v and beam current is 20mAand a signal voltage 40v. Find the input power, output power and efficiency.

Given-n=2, Ans -Io=20mA, VO=300v (1) Input power (Pac) = Io Vo  $= 20 \times 10^{-3} \times 300$ = 6 wPdc=6w (2) Output power (Pac) =  $\frac{2 Io Vo X' J1(X')}{2 \pi n - \frac{\pi}{2}}$  $=\frac{2\times20\times300\times2.048\times0.582}{2\pi\times2-\frac{\pi}{2}}$ .....where J1(X) =0.582 and X=1.841 = 1.36w Pac=1.36w (3) Efficiency (%) =  $\frac{Pac}{Pdc}$  $=\frac{1.36}{6} \times 100$ = 22.66%

Efficiency=22.66

Que.2)A reflex klystron operate under following condition - VO= 600v, L=1mm, Rash=15k $\Omega$ ,  $\frac{e}{m} = 1.759 \times 10^{11}$  and Fr = 9GHz. The tube oscillating at the peak of n =2 or 13/4mode assume that the transits time through the gap a beam loading can be neglected.

- (a) Find the value of repelled voltage (Ver.)
- (b) Find the direct current necessary to give microwave gap voltage of 200v
- (c) What is the electronic efficiency under this condition?

Ans- Given- VO = 600v,

L=1mm,

Rsh=15k $\Omega$ ,

 $\frac{e}{m} = 1.759 \times 10^{11},$ 

Fr = 9GHz.

(a) We know the relation between anode and repeller voltage,

$$\frac{Vo}{(Vo + Vr)^2} = \frac{(2 \pi n - \frac{\pi}{2})^2}{8 \omega^2 L^2} \frac{e}{m}$$

$$\frac{600}{(600 + Vr)^2} = \frac{(2 \pi \times 2 - \frac{\pi}{2})^2}{8 \times (2 \times \pi \times 9 \times 10^9)^2 \times (1 \times 10^{-3})^2} \times 1.759$$

$$(b) \text{ Assume, } \beta o = 1 \text{ (for perfect coupling)}$$

$$V2 = I2 \times Rsh$$

$$= 2 \times Io \times J1(X') \times Rsh$$

$$Io = \frac{200}{2 \times 0.52 \times 15 \times 10^3}$$

$$(c) \text{ Output power (Pac)} = \frac{2 Io Vo X' J1(X')}{2 \pi n - \frac{\pi}{2}}$$

$$=\frac{2\times12\times10^{-3}\times600\times2.048\times0.52}{2\times\pi\times2-\frac{\pi}{2}}$$

SSGMCE, Shegaon

(c)

```
= 1.75w
Input power (Pdc) = Io Vo

= 600 \times 12.82^{-3}
= 7.69w
Efficiency (%) = \frac{Pac}{Pdc}

= \frac{1.75}{7.69}
= 22.75\%
Efficiency=22.75\%
```

**Qu.3**) Reflex klystron operates at the 5GHz with an anode voltage of 1000v and cavity gap of 2mm. Calculate the gap transit time and the optimum length of drift region. Assume  $N = 1\frac{3}{4}$  and the Vr = -500v

```
Ans- Given- F= 5GHz,
```

VO=1000v,

d=2mm,

Vr=-500v,

n=2

$$VO = 0.593 \times 10^{6} \times \sqrt{Vo}$$
  
= 0.593 \times 10^{6} \times \sqrt{1000}  
= 18.75 Mv

$$\theta g = \omega \times \frac{d}{Vo}$$

 $= 2 \times \pi \times 5 \times 10^9 \times \frac{2 \times 10^{-3}}{18.75 \times 10^6}$ 

= 3.35rad

To find length of drift region,

$$\frac{Vo}{(Vo+Vr)} = \frac{(2\pi n - \frac{\pi}{2})^2}{8\omega^2 L^2} \frac{e}{m}$$

$$\frac{1000}{(1000 + (-500))} = \frac{(2\pi \times 2 - \frac{\pi}{2})^2}{8 \times (2 \times \pi \times 5 \times 10^9)^2 \times (L)^2} \times 1.759 \times 10^{11}$$

$$4 \times 10^{-3} = \frac{(2\pi \times 2 - \frac{\pi}{2})^2}{7.89 \times 10^{21} (L)^2} \times 1.759 \times 10^{11}$$

$$L=2.54 \text{mm}$$

**Que.4**) Reflex klystron is operated at 10GHz with a 600-beam voltage if the repeller voltage is 250v determine the repeller space for  $1\frac{3}{4}$  mode.

Ans-

$$\frac{Vo}{(Vo + Vr)^2} = \frac{(2\pi n - \frac{\pi}{2})^2}{8\omega^2 L^2} \frac{e}{m}$$

$$\frac{600}{(600 + 250)^2} = \frac{(2\pi \times 2 - \frac{\pi}{2})^2}{8 \times (2 \times \pi \times 10 \times 10^9)^2 \times (L)^2} \times 1.759 \times 10^{11}$$

$$4 \times 10^{-3} = \frac{(2\pi \times 2 - \frac{\pi}{2})^2}{7.89 \times 10^{21} (L)^2} \times 1.759 \times 10^{11}$$

**Que.5**)A reflex klystron operated at 9GHz with a dc beam current and voltage 30mA, 361v respectively repeller space is 0.1cm it is operated with  $1\frac{3}{4}$  mode. Determine the maximum output power and the operating repeller voltage.

**Ans-** Given- f= 9GHz,

Vo= 361v,

n=2,  
Io= 30mA,  
L=0.1cm,  

$$\frac{e}{m} = 1.759 \times 10^{11},$$

$$\frac{Vo}{(Vo + Vr)^2} = \frac{(2 \pi n - \frac{\pi}{2})^2}{8 \omega^2 L^2} \frac{e}{m}$$

$$\frac{361}{(361 + Vr)^2} = \frac{(2 \pi \times 2 - \frac{\pi}{2})^2}{8 \times (2 \times \pi \times 9 \times 10^9)^2 \times (1 \times 10^{-3})^2} \times 1.759$$

$$\boxed{Vr = 297v}$$
Output power (Pac) =  $\frac{2 lo Vo X' l(X')}{2 \pi n - \frac{\pi}{2}}$ 

$$= \frac{2 \times 30 \times 10^{-3} \times 361 \times 2.048 \times 0.582}{2 \times \pi \times 2 - \frac{\pi}{2}}$$
Pac = 2.34w

**Qu.6**) A reflex klystron is operating at 9GHz with a beam voltage of 250v and has the beam spacing of 0.5cm  $3\frac{3}{4}$  mode. Calculate the bandwidth when the repellervoltage change by 1v. **Ans-** we know that,

Pac=2.34w

$$\frac{Vo}{(Vo+Vr)^2} = \frac{(2\pi n - \frac{\pi}{2})^2}{8\omega^2 L^2} \frac{e}{m}$$
$$\frac{Vo}{(Vo+Vr)^2} = \frac{(n - \frac{1}{4})^2}{8f^2 L^2} \frac{e}{m}$$

Taking square on both the side,

$$\frac{1}{Vo + Vr} = \frac{(n - \frac{1}{4})}{2\sqrt{2} fL\sqrt{V0}} \sqrt{\frac{e}{m}}$$

$$Vr = \frac{-V0 + 2\sqrt{2} fL\sqrt{V0}}{\sqrt{\frac{e}{m}}(n - \frac{1}{4})}$$

$$Vr = \frac{-V0 + 6.744 \times 10^{-6} fL\sqrt{V0}}{(n - \frac{1}{4})}$$

$$Vr = \frac{-V0 + 6.744 \times 10^{-6} \times 0.5 \times 10^{-2} \frac{f}{\sqrt{V0}}}{3.75}$$

$$\Delta f = \frac{3.75}{6.744 \times 10^{-6} \times 0.5 \times 10^{-2} \sqrt{250}}$$

$$\Delta f = 7MHz$$

$$\Delta Vr = \frac{6.744 \times 10^{-6} \times 0.5 \times 10^{-2} \sqrt{250}}{3.75}$$

# **HELIX TRAVELLING WAVE TUBE (TWT)**

A helix travelling wave tube consists of an electron beam and a slow wave structure. The electron beam is focused by a constant magnetic field along the electron beam and the slow wave structure. This is termed an O-type travelling wave tube. The slow wave structure is either the helical type or folded backline.

#### III. <u>OPERATION</u>

The basic structure of TWT is as shown in the fig1. TWT consist of four major components generally assembled in configuration shown in fig1. The electron gun generates electron beam, which is confined by a distributed magnet system into the helix slow wave structure's beam tunnel. A helix like slow wave structure is used for the electron wave beam interaction and a collector at the end of the tube stops electron beam after it has travel through the tube.



Fig 9: schematic diagram of travelling wave tube (TWT)

#### \* <u>Slow wave structure:</u>

Several non-resonant periodic circuit or slow wave structure are designed for producing large gain over a wide bandwidth. Slow wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact.



Fig. 10: (a) Helical Coil (b) One turn of helix

Circumference of helix:

$$2\pi r = \frac{2\pi d}{2} = \pi d$$
$$z = \sqrt{p^2 + (\pi d)^2}$$

The ratio of the phase velocity VP along the pitch to the phase velocity along the coil is given by

$$\frac{\mathrm{Vp}}{c} = \frac{p}{\sqrt{p^2 + (\pi d)^2}} = \sin \Psi$$

Where,

c- Velocity of light

p -helix pitch

d - Diameter of helix

 $\Psi$  - Helix angle

### IV. <u>CHARACTERISTICS</u>

- Frequency range: 0.5GHz to 95GHz.
- Output power:
  - ➢ 5mW(10-40GHz): low power TWT
  - ▶ 250KW at 3GHz: high power TWT
  - ▶ 10MW (pulsed) at 3GHz

- Efficiency: 5-20%
- Noise figure:
  - ➢ 4-6dB (low power TWT 0.5 to 16GHz)
  - ➢ 25dB (low power TWT 0.5 to 16GHz

### V. <u>APLLICATIONS</u>

- Signal source in microwave generator.
- Local oscillators in receiver.
- Pump oscillators in parametric amplifier.
- FM oscillator in low power microwave links.

# **MAGNETRON**

An electron tube for amplifying or generating microwaves, with the flow of electrons controlled by an external magnetic field. The magnetron is a high-power vacuum tube that works as self-excited microwave oscillator. Crossed electron and magnetic field are used in the magnetron to produce the high-power output required in radar equipment.

#### I. CYLINDRICAL MAGNETRON

A schematic diagram of a cylindrical magnetron oscillator is shown in the fig1. This type of magnetron is also called as conventional magnetron.

In a cylindrical magnetron, several re-entrant cavities are connected to the gaps. The DC voltage Vo is applied between the cathode and the anode. The magnetic flux density Vo is in the positive z direction. When the DC voltage and the magnetic flux are adjusted properly, the electrons will follow cycloid paths in the cathode-anode space under the combine force of both electric and magnetic field as shown in fig2.



Fig. 11: schematic diagram of a cylindrical magnetron



Fig.12: Electron path in a cylindrical magnetron

#### Expression for hull cut of magnetic field and voltage equation

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}Er - \frac{e}{m}rBz\frac{d\phi}{dt}$$
$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}Bz\frac{dr}{dt}$$

Where  $\frac{e}{m} = 1.759 \times 10^{11} \frac{C}{Kg}$  is the charge to mass ratio of the electron and Bo=Bz is assumed in the positive z direction.

$$\frac{d}{dt}r^{2} = 2r\frac{dr}{dt}$$
$$\frac{1}{2}\frac{d}{dt}r^{2} = r\frac{dr}{dt}$$
$$\frac{d}{dt}\left(r^{2}\frac{d\phi}{dt}\right) = \frac{1}{2}\frac{e}{m}Bz\frac{d}{dt}r^{2}$$
$$\frac{d}{dt}\left(r^{2}\frac{d\phi}{dt}\right) = \frac{1}{2}\omega c\frac{d}{dt}r^{2}$$

Here taking Integration....

$$r^2 \frac{d\phi}{dt} = \frac{1}{2}\omega c r^2 + const.$$

At r=a; where a is radius of the cathode cylinder,

 $\frac{d\phi}{dt} = 0$ ; const= $\frac{-1}{2}\omega c a^2$ . The angular velocity is expressed by  $r^2 \frac{d\phi}{dt} = \frac{1}{2}\omega c r^2 - \frac{1}{2}\omega c a^2$ 

$$= \frac{1}{2}\omega c(r^{2} - a^{2})$$
$$\frac{d\phi}{dt} = \frac{1}{2}\omega c\left(1 - \frac{a^{2}}{r^{2}}\right)....(1)$$

Since the magnetic field does not work on the electrons, Kinetic energy of electron is given by,

$$\frac{1}{2}mv^{2} = eV$$

$$v^{2} = 2\frac{eV}{m}$$

$$V^{2} = Vr^{2} + V\phi^{2}$$

$$= 2\frac{eV}{m} = \left(\frac{dr}{dt}\right)^{2} + \left(r\frac{d\phi}{dt}\right)^{2}$$

Taking square root,

$$b\left[\frac{1}{2}\omega c\left(1-\frac{a^2}{b^2}\right)\right] = \left(2\frac{eVo}{m}\right)^{1/2}$$
$$\frac{b\omega cb^2 - b\omega ca^2}{2b^2} = \left(2\frac{eVo}{m}\right)^{1/2}$$
$$Bo = \frac{\left(2\frac{eVo}{m}\right)^{1/2} \times 2b^2 \times \frac{m}{e}}{b^3\left(1-\frac{a^2}{b^2}\right)}$$

$$Boc = \frac{\left(8\frac{m}{e}Vo\right)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$$

This means that if *B*o>Boc for given *V*o, the electron will not reach the anode. The electron will not reach the anode. Conversely,thecutoff voltage is given by, Now, squaring both the side;

$$Bo^{2} = \frac{8\frac{m}{e}Vo}{b^{2}\left(1-\frac{a^{2}}{b^{2}}\right)^{2}}$$
$$Voc = \frac{Bo^{2}b^{2}\left(1-\frac{a^{2}}{b^{2}}\right)^{2}}{8\frac{m}{e}}$$
$$Voc = \frac{e}{8m}Bo^{2}b^{2}\left(1-\frac{a^{2}}{b^{2}}\right)^{2}$$

This means that if Vo<Voc for a given Bo the electron will not reach the anode. Then this equation is called Hull cut-off voltage equation.

# **MAGNETRON PARAMETERS AND NUMERICALS**

### Typical magnetron impedance characteristics

Some of the following parameters are often found in magnetron specifications.



## 1). Mode jump:

Change in mode of magnetron operation from 1 pulse to the next each mode represents different undesirables to avoids. We have to use the solution is strapped magnetron.

## 2). Strapping:

A multicavity magnetron in which resonator segment having the same polarity are connected together by small conducting strips to supress undesired modes of oscillations.

### **3). Frequency pushing:**

The oscillating frequency is affected by the electron density in the interaction space of the magnetron – this is a function of the anode current. If the top of the current pulse is not flat, this will result in modulation of the frequency as well as of the power level.



Fig1. Typical frequency pushing curve for 10 kW 3rd generation marine magnetrons (MG 5241).

The datasheets for some types include maximum limits on frequency pushing, expressed in MHz/A (megahertz per ampere) over a specified current range. Unless otherwise specified, the frequency pushing is measured with the magnetron feeding a matched load, and can be greater under mismatched conditions.

## 4). Frequency pulling& pushing

Frequency variation of magnetron due to changes in mode impendence is called as frequency pulling. Similar to reflex klystron, it is possible to change the resonance frequency of magnetron by changing anode voltage. This refer to as frequency pushing, is due the fact that the change in orbital velocity of electrons.

This is a measure of change of frequency with change of phase of load mismatch, and it is clearly desirable to minimize this characteristic in most magnetrons. The pulling figure is usually

defined as the maximum frequency change when a fixed external mismatch (usually 1.5:1 VSWR but sometimes 1.3:1 VSWR) is moved one half wavelength in the output waveguide.

# Numerical:

An X-band pulsed cylindrical magnetron has the following operating parameters:

Anode voltage:	Vo=26kV
Beam current:	Io = 27A
Magnetic flux density:	Bo=0.336Wb/m <sup>2</sup>
Radius of cathode cylin	der: a=5cm

Radius of vane edge to centre: b=10cm

#### Compute:

- a. The cyclotron angular frequency
- b. The cut off voltage for a fixed Bo
- c. The cut off magnetic flux density for fixed Vo

#### Solution:

- a. The cyclotron angular frequency is  $\omega c = \frac{e}{m} Bo = 1.759 \times 10^{11} \times 0.336 = 5.91 \times 10^{10} rad$
- b. The cut off voltage for fixed Bo is

$$Voc = \frac{e}{8m} Bo^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$
$$= \frac{1}{8} \times 1.795 \times 10^{11} \times (0.336)^2 (10 \times 10^{-2}) \times (10 \times 10^{-2}) \times \left(1 - \frac{5^2}{10^2}\right)^2$$
$$= 13.96 \text{MV}$$

c. The cut off magnetic flux density for fixed Vo is

$$Boc = \frac{\left(8\frac{m}{e}Vo\right)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$$
$$= (8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}})^{\frac{1}{2}} \times \left[10 \times 10^{-2} \left(1-\frac{5^2}{10^2}\right)\right]^{-1}$$
$$= 14.495 \text{mWb}/m^2$$